



**Rotterdam, The Netherlands**  
July 26<sup>th</sup>, 2010



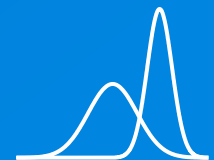
# ***Chi Square Control Charts***

**Presented By:**

**Dr. John M. Ennis**

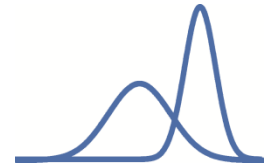
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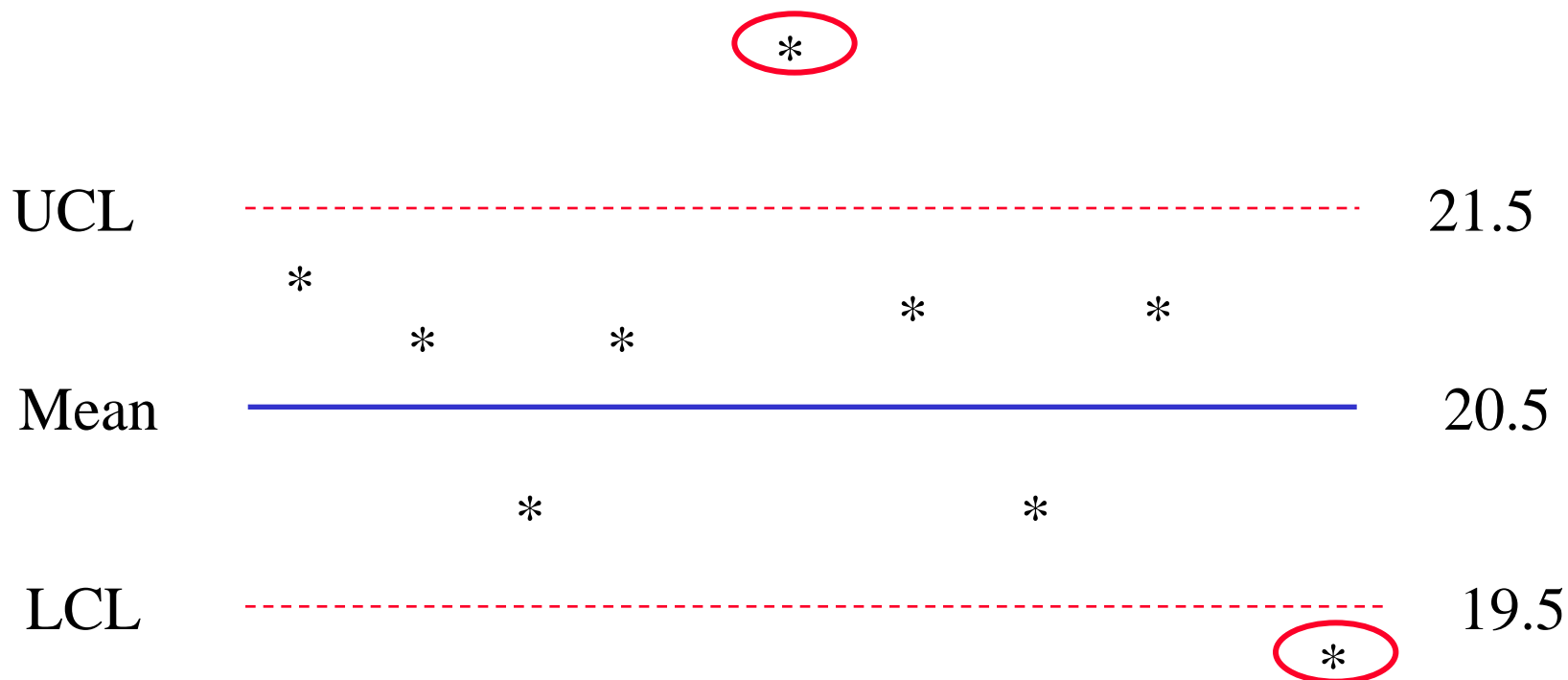




# An Image

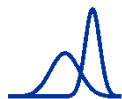


# Mean Control Chart



Mean: 20.5

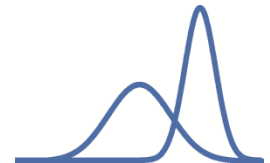
Standard Error of the Mean: 0.5





# Multiple Sensory Descriptors

- Descriptive and consumer panel ratings data
- Highly multivariate
- Correlated assessments
- Assumption of homogenous assessors
  - ❖ Test with Dirichlet Multinomial model

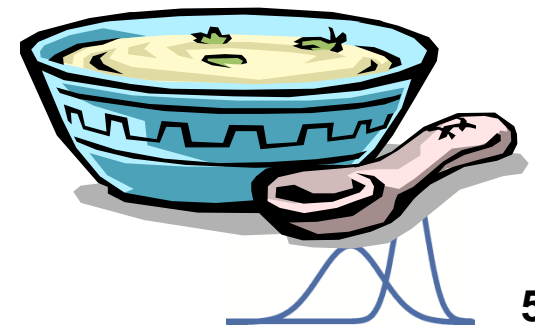




# Soup Example

- Existing data for current production soup ratings and ratings for a test sample

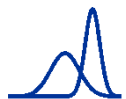
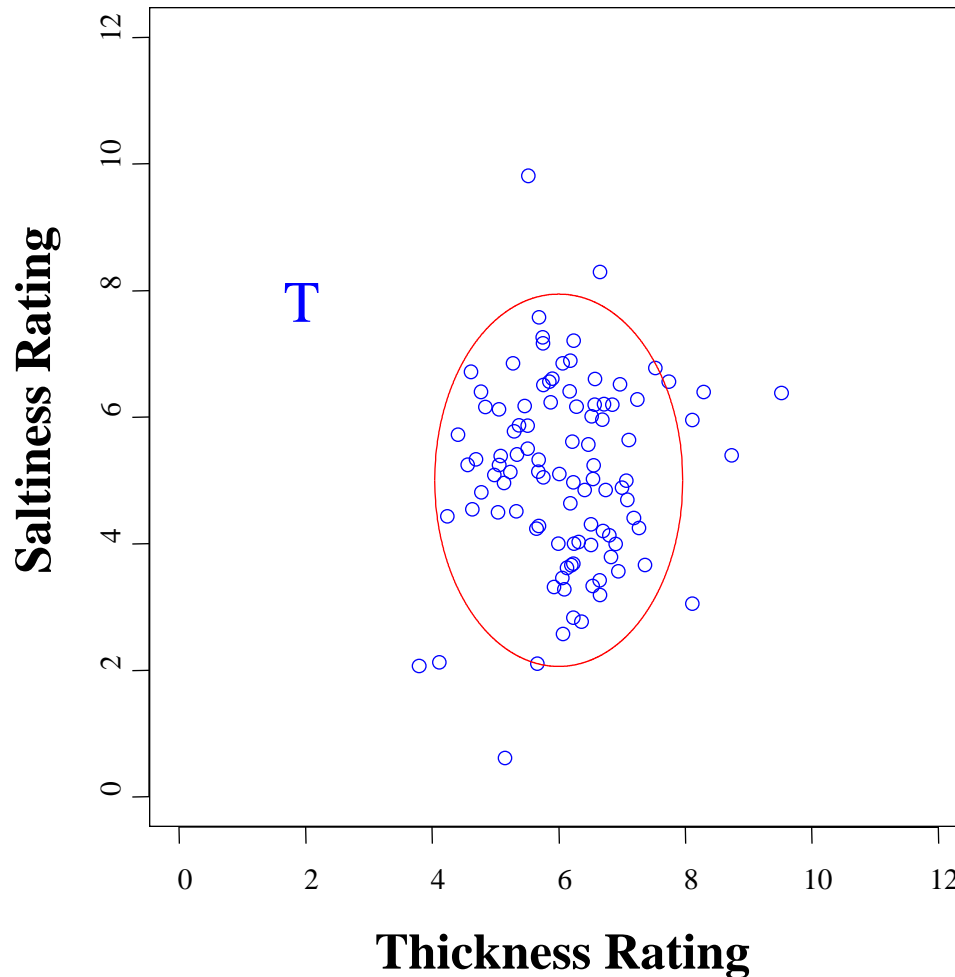
Product	Thickness	Saltiness
Current	4.1	4.5
Current	5.3	5.0
Current	6.6	3.8
...	...	...
Current	4.2	5.3
Test Sample	2.0	8.0





# Bivariate Normal Control Chart

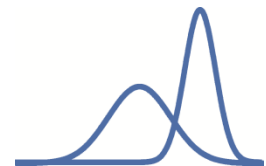
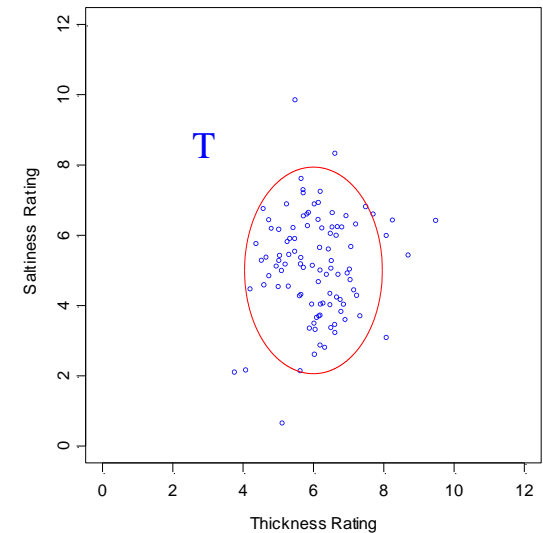
(o = current production, T = test sample)





# Comments

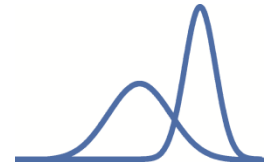
- Product X rated low on thickness and high on saltiness
- 95% contour for the bivariate normal suggests it not typical
- Can we get a measure of departure from the current process?





# Beverage Example

- Ratings on sweetness, sourness and carbonation available for current production and a test product
- Is the test product within specification?
- Construct a star plot with confidence limits



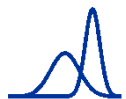


# Star Plot Control Chart

Sweetness

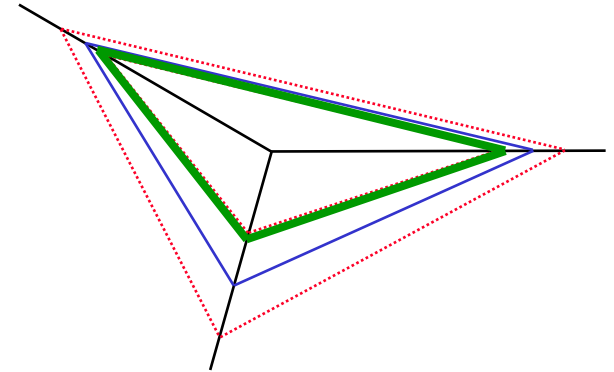
Carbonation

Sourness

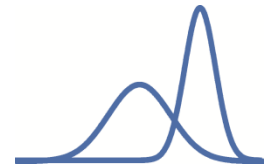




# Comments



- Star plots are univariate plots of more than one attribute
- No account made of attribute correlations or the multivariate nature of the data
- Questions:
  - ❖ Product inside acceptance region means it should be accepted?
  - ❖ Product outside acceptance region means it should be rejected?





# Conversion to Standard Form

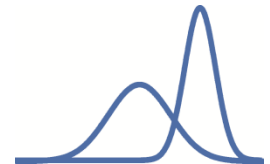
- For univariate normal variables,

$$Z = \frac{X - \mu}{\sigma}$$

- The analogous Cholesky transformation exists for multivariate normal variables

$$\mathbf{z} = \mathbf{A}^{-1} (\mathbf{x} - \boldsymbol{\mu})$$

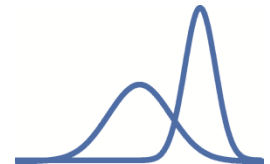
$$\mathbf{A}\mathbf{A}^t = \mathbf{V}$$





# Why Transform the Data?

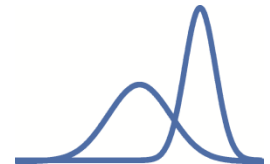
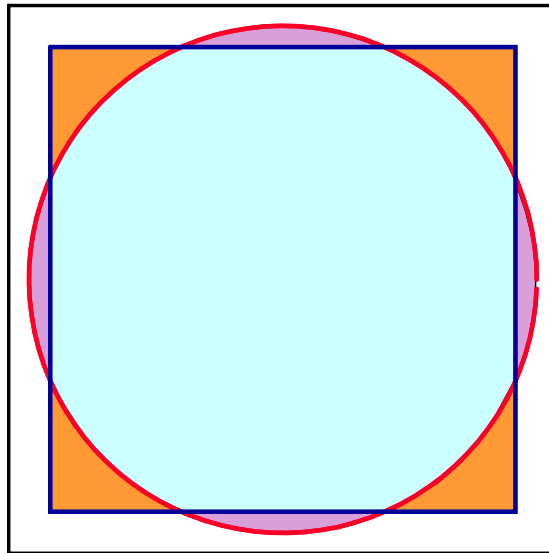
- Standard distributions desired for testing
- Similar to calculating z-scores and using normal tables
- In standard form results are more meaningful and easier to interpret



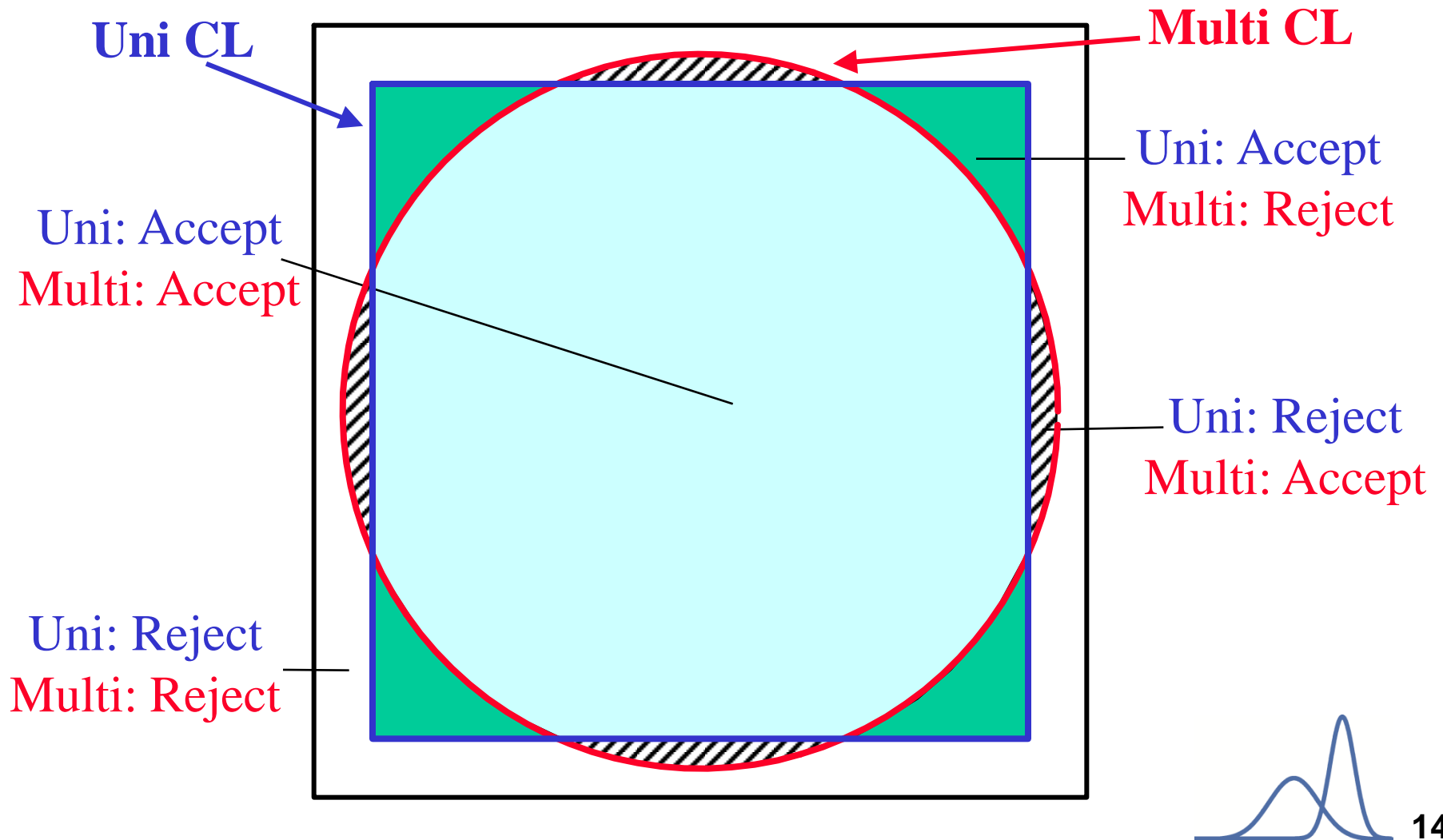


# Equal Likelihood Contours

- For a non standard normal, contours of equal likelihood are ellipses
- For a standard normal, contours are circles
- Points falling outside these contours are not typical

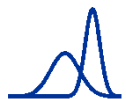
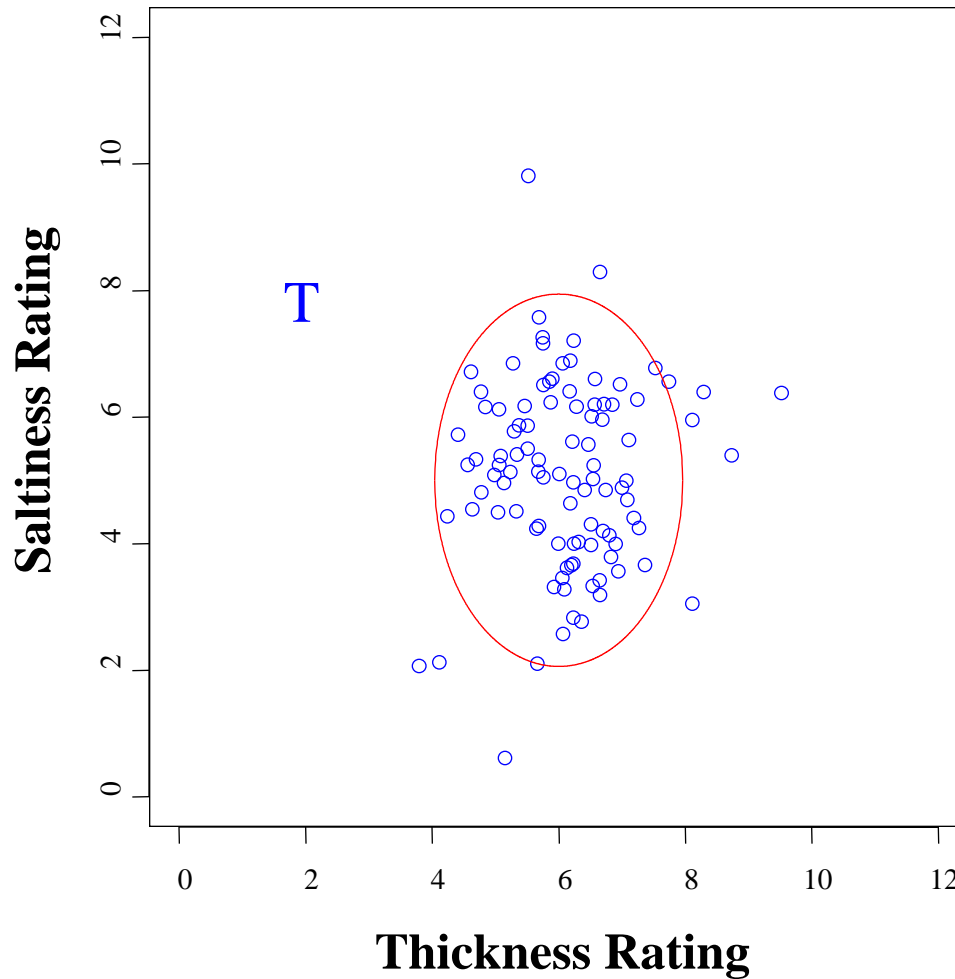


# Acceptance Regions



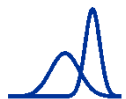
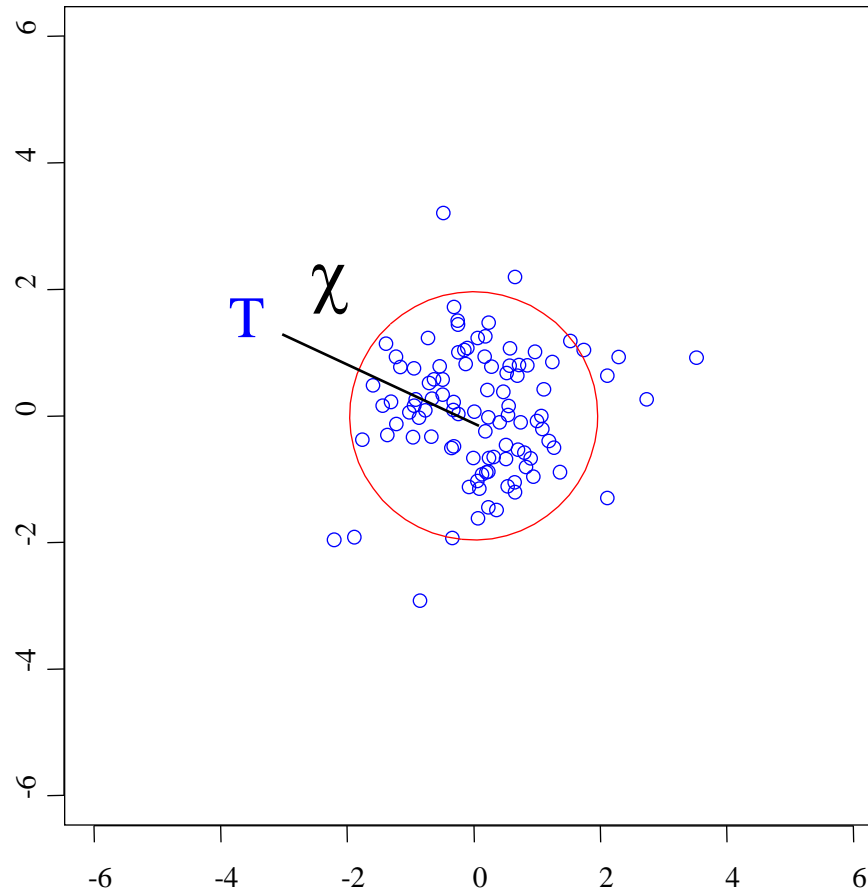


# Bivariate Normal Control Chart





# Becomes...

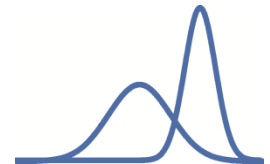






# Statistics: Multivariate vs Univariate

- Products may fail a multivariate QC test and pass a univariate test or the opposite
- Need to take into account correlations among attributes
- Need to take into account accumulated evidence of deviance from a standard





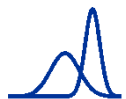
# Beverage Example

Sweetness

Sourness

Carbonation

- Although acceptable on each attribute,  $\chi^2$  test rejects this sample





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